Mathematic: Applications & Interpretation SL & HL

Formula Sheet−First Examinations 2021

Prior Learning SL & HL

Area: Parallelogram	$A = bh$, b = base, h = height
Area: Triangle	$A = \frac{1}{2}bh$, b = base, h = height
Area: Trapezoid	$A = \frac{1}{2}(a+b)h$, a, b = parallel sides, h = height
Area: Circle	$A = \pi r^2$, r = radius
Circumference: Circle	$C = 2\pi r$, r = radius
Volume: Cuboid	$V = lwh$, l = length, w = width, h = height
Volume: Cylinder	$V = \pi r^2 h$, r = radius, h = height
Volume: Prism	$V = Ah$, A = cross-section area, h = height
Area: Cylinder curve	$A = 2\pi rh$, r = radius, h = height
Distance between 2 points $(x_1, y_1), (x_1, y_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of midpoint of a line with endpoints $(x_1, y_1), (x_2, y_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Prior Learning HL only

Solution of a quadratic equation in the form $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

Topic1: Number and algebra- SL & HL

The nth term of an arithmetic sequence	$a_n = a_1 + (n-1)d$
Sum of n term of an arithmetic sequence	$s_n = \frac{n}{2}(a_1 + a_n)$
The nth term of an geometric sequence	$a_n = a_1 r^{n-1}$
Sum of n terms of a finite geometric sequence	$s_n = \frac{a_1(r^n - 1)}{r - 1}$ $r \neq 1$
Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ FV = future value, PV = present value, n = the number of years, k = the number of compounding periods per year, $r\%$ = the annual rate of interest
Exponents & logarithms	$a^x = b \Leftrightarrow x = \log_a b$, $a, b > 0, a \neq 1$
Percentage error	$\varepsilon = \left\lvert\frac{v_A - v_E}{v_E}\right\rvert \times 100\%$ v_A = approximate value, v_E = exact value

Topic 1: Number and algebra – HL only

Law of logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^n = n \log_a x$
The sum of an infinite geometric sequence	$s_\infty = \frac{a_1}{1 - r}$, $\lvert r\rvert < 1$
Complex numbers	$z = a + bi$
Discriminant	$\Delta = b^2 - 4ac$
Modulus-argument (polar) & Exponential (Euler) form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$
Determinant of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A = \lvert A\rvert = ad - bc$
Inverse of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\Rightarrow A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
Power formula for a matrix	$A^n = PD^nP^{-1}$, where P is the matrix of eigenvectors and D is the diagonal matrix of eigenvalues

Topic 2: Functions- SL &HL

Equations of a straight line	$y = mx + b$; $y - y_0 = m(x - x_0)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow x = -\frac{b}{2a}$

Topic 2: Functions- HL only

Logistic function	$f(x) = \frac{L}{1 + Ce^{-kx}}$, $L, k, C > 0$

Topic 3: Geometry and trigonometry - SL & HL

Distance between 2 points $(x_1, y_1, z_1), (x_2, y_2, z_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of midpoint of a line with endpoints $(x_1, y_1, z_1), (x_2, y_2, z_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Volume: Right-pyramid	$V = \frac{1}{3}Ah$, A = base area, h = height
Volume: Right cone	$V = \frac{1}{3}\pi r^2 h$, r = radius, h = height
Area: Cone curve	$A = \pi rl$, r = radius, l = slant height
Volume: Sphere	$V = \frac{4}{3}\pi r^3$, r = radius
Surface area: Sphere	$A = 4\pi r^2$, r = radius
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
Area: Triangle	$A = \frac{1}{2}ab\sin C$
Length of an arc	$l = \frac{\theta}{360} \cdot 2\pi r$, θ = angle in degrees, r = radius
Area of a sector	$A = \frac{\theta}{360} \cdot \pi r^2$, θ = angle in degrees, r = radius

Topic 3: Geometry and trigonometry- HL only

Length of an arc	$l = r\theta$, r = radius, θ = angle in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$, r = radius, θ = angle in radians
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
Identity for $\tan\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
Transformation matrices	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$: reflection in the line $y = (\tan\theta)x$ $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$: clockwise rotation of angle θ about $(0,0)$ $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$: horizontal stretch by scale factor of k $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$: vertical stretch with scale factor of k $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, centre $(0,0)$: enlargement with scale factor of k
Magnitude of a vector	$\lvert v\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
Vector equation of a line	$r = a + \lambda b$
Parametric form of the equation of a line	$x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$
Scalar product	$v \cdot w = v_1w_1 + v_2w_2 + v_3w_3$ $v \cdot w = \lvert v\rvert\lvert w\rvert\cos\theta$, where θ is the angle between v and w
Angle between two vectors	$\cos\theta = \frac{v \cdot w}{\lvert v\rvert\lvert w\rvert}$
Vector product	$v \times w = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$ $\lvert v \times w\rvert = \lvert v\rvert\lvert w\rvert\sin\theta$, where θ is the angle between v and w
Area of a parallelogram	$A = \lvert v \times w\rvert$, where v and w form two adjacent sides of a parallelogram

Topic 4: Statistics and probability- SL & HL

Interquartile range	$\text{IQR} = Q_3 - Q_1$
Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{1}{n}\sum_{i=1}^{k} w_i x_i$, where $n = \sum_{i=1}^{k} w_i$
Probability of an event A	$\text{P}(A) = \frac{n(A)}{n(S)}$
Complementary events	$\text{P}(A) + \text{P}(A') = 1$
Combined events	$\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$
Mutually exclusive events	$\text{P}(A \cup B) = \text{P}(A) + \text{P}(B)$
Conditional probability	$\text{P}(A\lvert B) = \frac{\text{P}(A \cap B)}{\text{P}(B)}$
Independent events	$\text{P}(A \cap B) = \text{P}(A)\text{P}(B)$
Expected value: Discrete random variable X	$\text{E}(X) = \sum x\,P(X = x)$
Binomial distribution Mean; Variance	$X \sim \text{B}(n, p)$; $\text{E}(X) = np$; $\text{Var}(X) = np(1-p)$

Topic 4: Statistics and probability- HL only

Linear transformation of a single r. v. X	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{Var}(aX + b) = a^2\text{Var}(X)$
Linear combinations of n independent random variables, $X_1, X_2, \ldots, X_n$	$\text{E}(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)$ $= a_1\text{E}(X_1) \pm a_2\text{E}(X_2) \pm \ldots \pm a_n\text{E}(X_n)$ $\text{Var}(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)$ $= a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2) + \cdots$ $+ a_n^2\text{Var}(X_n)$
Unbiased estimated of population variance	$s_{n-1}^2 = \frac{n}{n-1}s_n^2$
Poisson distribution	$X \sim \text{Po}(\lambda)$; $\text{E}(X) = \lambda$; $\text{Var}(X) = \lambda$
Transition matrices	$T^n s_0 = s_n$, where s_0 is the initial state.

Topic 5: Calculus – SL & HL

Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
Integral of x^n	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
Area between curve $y = f(x)$ & x-axis	$A = \int_a^b y dx$, where $f(x) > 0$
The trapezoidal rule, $h = \frac{b-a}{n}$	$\int_a^b y dx \approx \frac{1}{2}h(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$

Topic 5: Calculus- HL only

Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u), u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Product rule	$y = uv \Rightarrow y' = u'v + uv'$
Quotient rule	$y = \frac{u}{v} \Rightarrow y' = \frac{u'v - uv'}{v^2}$
Standard integrals	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$ $\int \sin x\ dx = -\cos x + C$ $\int \cos x\ dx = \sin x + C$ $\int e^x dx = e^x + C$
Area enclosed by a curve and x or y-axes	$A = \int_a^b \lvert y\rvert dy$ or $A = \int_a^b \lvert x\rvert dy$
Volume of revolution about x or y-axes	$V = \int_a^b \pi y^2\ dx$ or $V = \int_a^b \pi x^2\ dy$
Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
Distance; Displacement travelled from t_1 to t_2	$\text{dist} = \int_{t_1}^{t_2} \lvert v(t)\rvert\,dt$; $\text{disp} = \int_{t_1}^{t_2} v(t)\,dt$
Euler's method	$y_{n+1} = y_n + hf(x_n, y_n)$; $x_{n+1} = x_n + h$, where h is a constant (step length)
Euler's method for coupled systems	$x_{n+1} = x_n + hf_1(x_n, y_n, t_n)$; $y_{n+1} = y_n + hf_2(x_n, y_n, t_n)$; $t_{n+1} = t_n + h$, where h is a constant (step length)
Exact solution for coupled linear DE	$x = Ae^{\lambda_1 t}P_1 + Be^{\lambda_2 t}P_2$