Mathematics: Analysis & Approaches SL & HL

Formula Sheet—First Examinations 2021

Prior Learning SL & HL

Area: Parallelogram	$A = bh$, $b =$ base, $h =$ height
Area: Triangle	$A = \frac{1}{2}bh$, $b =$ base, $h =$ height
Area: Trapezoid	$A = \frac{1}{2}(a+b)h$, $a, b =$ parallel sides, $h =$ height
Area: Circle	$A = \pi r^2$, $r =$ radius
Circumference: Circle	$C = 2\pi r$, $r =$ radius
Volume: Cuboid	$V = lwh$, $l =$ length, $w =$ width, $h =$ height
Volume: Cylinder	$V = \pi r^2 h$, $r =$ radius, $h =$ height
Volume: Prism	$V = Ah$, $A =$ cross-section area, $h =$ height
Area: Cylinder curve	$A = 2\pi rh$, $r =$ radius, $h =$ height
Distance between 2 points $(x_1, y_1), (x_2, y_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of midpoint of a line with endpoints $(x_1, y_1), (x_2, y_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Topic1: Number and algebra- SL & HL

The nth term of an arithmetic sequence	$a_n = a_1 + (n-1)d$
Sum of n term of an arithmetic sequence	$s_n = \frac{n}{2}(a_1 + a_n)$
Sum of n terms of a finite geometric sequence	$s_n = \frac{a_1(r^n - 1)}{r - 1}$ $r \neq 1$
Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ $FV =$ future value, $PV =$ present value, $n =$ the number of years, $k =$ the number of compounding periods per year, $r\% =$ the annual rate of interest
Exponents & logarithms	$a^x = b \Leftrightarrow x = \log_a b$, $a, b > 0, a \neq 1$
Exponents & logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^n = n \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
The sum of an infinite geometric sequence	$s_\infty = \frac{a_1}{1 - r}$, $\lvert r \rvert < 1$
Binomial Theorem for $n \in \mathbb{N}$, $(a+b)^n =$	$a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$
Binomial coefficient	$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$

Topic 1: Number and algebra – HL only

Combinations; Permutations	${}^nC_r = \frac{n!}{r!\,(n-r)!}$; ${}^nP_r = \frac{n!}{(n-r)!}$
Newton's theorem, $n \in \mathbb{Q}$, $(a+b)^n =$	$a^n\left(a^n + n\left(\frac{b}{a}\right) + \frac{n(n-1)}{2!}\left(\frac{b}{a}\right)^2 + \cdots\right)$
Complex numbers	$z = a + bi$
Modulus-argument (polar) & Exponential (Euler) form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$
De Moivre's theorem	$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Topic 2: Functions- SL &HL

Equations of a straight line	$y = mx + b$; $y - y_0 = m(x - x_0)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow x = -\frac{b}{2a}$
Solution of a quadratic equation in the form $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$
Discriminant	$\Delta = b^2 - 4ac$
Exponential and logarithmic functions	$a^x = e^{x \ln a}$; $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$

Topic 2: Functions- HL only

Vieta's Formula (Sum and product of the roots of polynomial)	$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$ Sum $= \frac{-a_{n-1}}{a_n}$; product $= \frac{(-1)^n a_0}{a_n}$

Topic 3: Geometry and trigonometry - SL & HL

Distance between 2 points $(x_1, y_1, z_1), (x_2, y_2, z_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of midpoint of a line with endpoints $(x_1, y_1, z_1), (x_2, y_2, z_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Volume: Right-pyramid	$V = \frac{1}{3}Ah$, $A =$ base area, $h =$ height
Volume: Right cone	$V = \frac{1}{3}\pi r^2 h$, $r =$ radius, $h =$ height
Area: Cone curve	$A = \pi rl$, $r =$ radius, $l =$ slant height
Volume: Sphere	$V = \frac{4}{3}\pi r^3$, $r =$ radius
Surface area: Sphere	$A = 4\pi r^2$, $r =$ radius
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
Area: Triangle	$A = \frac{1}{2}ab\sin C$
Length of an arc	$l = r\theta$, $r =$ radius, $\theta =$ angle in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$, $r =$ radius, $\theta =$ angle in radians
Identity for $\tan\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
Double angle identities	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Topic 3: Geometry and trigonometry- HL only

Reciprocal trigonometric identities	$\sec\theta = \frac{1}{\cos\theta}$; $\csc\theta = \frac{1}{\sin\theta}$
Pythagorean identities	$1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \csc^2\theta$
Compound angle identities	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
Double angle identity for tan	$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$
Magnitude of a vector	$\lvert v \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
Scalar product	$v \cdot w = v_1w_1 + v_2w_2 + v_3w_3$ $v \cdot w = \lvert v \rvert \lvert w \rvert \cos\theta$, where θ is the angle between v and w
Angle between two vectors	$\cos\theta = \frac{v \cdot w}{\lvert v \rvert \lvert w \rvert}$
Vector equation of a line	$r = a + \lambda b$
Parametric form of the equation of a line	$x = x_0 + \lambda l$, $y = y_0 + \lambda m$, $z = z_0 + \lambda n$
Cartesian equations of a line	$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
Vector product	$v \times w = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$ $\lvert v \times w \rvert = \lvert v \rvert \lvert w \rvert \sin\theta$, where θ is the angle between v and w
Area of a parallelogram	$A = \lvert v \times w \rvert$, where v and w form two adjacent sides of a parallelogram
Vector equation of a plane	$r = a + \lambda b + \mu c$
Equation of a plane	$r \cdot n = a \cdot n$ (using the normal vector)
Cartesian equ. of a plane	$ax + by + cz = d$

Topic 4: Statistics and probability- SL & HL

Interquartile range	$\mathrm{IQR} = Q_3 - Q_1$
Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{1}{n}\sum_{i=1}^{k} w_i x_i$, where $n = \sum_{i=1}^{k} w_i$
Probability of an event A	$\mathrm{P}(A) = \frac{n(A)}{n(S)}$
Complementary events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
Combined events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
Mutually exclusive events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
Conditional probability	$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
Independent events	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$
Expected value: Discrete random variable X	$\mathrm{E}(X) = \sum x\,P(X = x)$
Binomial distribution Mean; Variance	$X \sim \mathrm{B}(n, p)$; $\mathrm{E}(X) = np$; $\mathrm{Var}(X) = np(1-p)$
Standard score	$z = \frac{x - \mu}{\sigma}$

Topic 4: Statistics and probability- HL only

Bayes' theorem	$P(B \mid A) = \frac{P(B)P(A \mid B)}{P(B)P(A \mid B) + P(B')P(A \mid B')}$ $P(B_i \mid A) = \frac{P(B_i)P(A \mid B)}{P(B_i)P(A \mid B_i) + P(B_2)P(A \mid B_2) + P(B_3)P(A \mid B_3)}$
Variance σ^2	$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \mu^2$
Standard Deviation σ	$\sigma = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(x_i - \mu)^2}$
Linear transformation of a single r. v. X	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$
Expected value: Continuous r. v. X	$\mathrm{E}(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx$
Variance	$\mathrm{Var}(X) = \mathrm{E}[(X - \mu)^2] = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$
Variance of a discrete random variable X	$\mathrm{Var}(X) = \sum (x - \mu)^2 f(x) = \sum x^2 f(x) - \mu^2$
Variance of a continuous random variable X	$\mathrm{Var}(X) = \int_{-\infty}^{\infty}(x - \mu)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

Topic 5: Calculus – SL & HL

Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
Integral of x^n	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
Area between curve $y = f(x)$ & x-axis	$A = \int_a^b y\,dx$, where $f(x) > 0$
Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u), u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Product rule	$y = uv \Rightarrow y' = u'v + uv'$
Quotient rule	$y = \frac{u}{v} \Rightarrow y' = \frac{u'v - uv'}{v^2}$
Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
Distance; Displacement travelled from t_1 to t_2	dist $= \int_{t_1}^{t_2} \lvert v(t) \rvert\,dt$; disp $= \int_{t_1}^{t_2} v(t)\,dt$
Standard integrals	$\int \frac{1}{x}dx = \ln\lvert x \rvert + C$ $\int \sin x\,dx = -\cos x + C$ $\int \cos x\,dx = \sin x + C$ $\int e^x dx = e^x + C$
Area enclosed by a curve and x-axis	$A\int_a^b \lvert y \rvert dx$

Topic 5: Calculus- HL only

Derivative of $f(x)$	$\frac{dy}{dx} = f'(x) = \lim_{h \to 0}\frac{f(x+h) - f(x)}{h}$
Standard derivatives	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$ $f(x) = \sec x \Rightarrow f'(x) = \sec x\tan x$ $f(x) = \csc x \Rightarrow f'(x) = -\csc x\cot x$ $f(x) = \cot x \Rightarrow f'(x) = -\csc^2 x$ $f(x) = a^x \Rightarrow f'(x) = a^x\ln a$
Standard integrals	$\int a^x\,dx = \frac{1}{\ln a}a^x + C$ $\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}}dx = \arcsin\left(\frac{x}{a}\right) + C$, $\lvert x \rvert < a$
Integration by parts	$\int uv'\,dx = uv - \int u'v\,dx$
Area enclosed by a curve and x or y-axes	$A = \int_a^b \lvert y \rvert\,dy$ or $A = \int_a^b \lvert x \rvert\,dy$
Volume of revolution about x or y-axes	$V = \int_a^b \pi y^2\,dx$ or $V = \int_a^b \pi x^2\,dy$
Euler's method	$y_{n+1} = y_n + hf(x_n, y_n)$; $x_{n+1} = x_n + h$, where h is a constant (step length)
Integrating factor for $y' + P(x)y = f(x)$	$e^{\int P(x)dx}$
Maclaurin series	$f(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \cdots$
Maclaurin series for special functions	$e^x = 1 + x + \frac{x^2}{2!} + \cdots, x \in \mathbb{R}$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots, x \in \mathbb{R}$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots, x \in \mathbb{R}$ $\frac{1}{1-x} = 1 + x + x^2 + \cdots, -1 < x < 1$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots, -1 < x \leq 1$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots, -1 \leq x \leq 1$