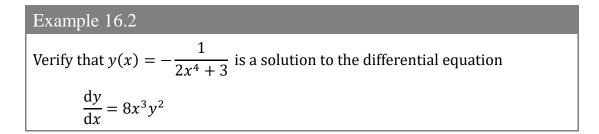
Chapter 16: More integral calculus

Example 16.1 Verify that $y(x) = Ce^{x^4}$ is a solution to the differential equation $\frac{dy}{dx} = 4x^3y$



Example 16.3

Find a solution to the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x+2}, \quad x \neq -2$$

Example 16.4

Find the general solution of the differential equation $(1 - 2)^2 = \frac{1}{2} - \frac{1}{2}$

 $y' - 9x^2y^2 = 5y^2$

Example 16.5	
Solve the differential equation	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x^2y}{1+4y^2}$	

Example 16.6 Find the general solution of the differential equation $x^{2}y\frac{dy}{dx} = x + 1, \quad x > 0, \quad y > 0$

Example 16.7 Solve the initial value problem $\frac{dy}{dx} = \frac{y}{x+1}; \ y(1) = 8$

Example 16.8

Solve the initial value problem

$$\frac{dy}{dt} = (e^{y-t})\frac{1+t^2}{\cos y}; \ y(0) = 0$$

Example 16.9

Solve the differential equation $2xdx + e^{x+y} \cos y \, dy = 0$

Example 16.10

Find the general solution of the population growth model

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$$

Example 16.11

A cold object is placed in warmer medium that is kept at a constant temperature S. The rate of change of the temperature T(t) with respect to time is proportional to the difference between the surrounding medium and the object and hence it satisfies

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(S - T) \text{ and } T(0) = T_0$$

where k > 0 and $T_0 < S$; that is, the initial temperature is less than the temperature of the surrounding medium. Find the solution to the initial value problem.

Example 16.12

The rate of decay of a substance y at any time t is directly proportional to the amount of y and also directly proportional to the amount of another substance x.

The constant of proportionality is $-\frac{1}{2}$ and the value of *x* at any time

t is given by $x = \frac{4}{(1+t)^2}$

- (a) Given the initial conditions that y = 10 when t = 0, find y as an explicit function of t.
- (b) Determine the amount of the substance remaining as t becomes very large.

Example 16.13

Find the general solution to the differential equation $\frac{dy}{dx} = 2xy$

Example 16.14

Solve the logistic differential equation $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$

Example 16.15

An official wildlife commission releases 20 deer into a protected forest.

After 10 years, the deer population is 208. The commission believes that the environment cannot support more than 400 deer.

- (a) Write a model for the deer population in terms of the number of years, t.
- (b) Use the model to estimate the deer population after 10 years.
- (c) Discuss the long-term trend of the deer population.

Example 16.16

Solve the differential equation $xyy' = x^2 + y^2$

Example 16.17

Find the general solution of $(x^2 - y^2)dx + xydy = 0$

Example 16.18	
Find all solutions to	
$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = x$	

Example 16.19 Find all solutions to $2x \frac{dy}{dx} + 8y = 7x^4$

Example 16.20

Find the initial value solution of $y' + y \tan x = \cos^2 x$, y(0) = 1

Example	16.21

Find the solution to:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{2t}{1+t^2}y + \frac{2}{1+t^2}, \qquad y(0) = 0$$

Example 16.22 Solve the following initial value problem for x > 0 $2x \frac{dy}{dx} - y = 3x + 1, \qquad y(3) = 4$

Example 16.23

A tank contains 50 litres of a solution composed of 90% water and 10% hydrochloric acid. A second solution containing 50% water and 50% hydrochloric acid is being poured into the tank at a rate of 4 litres per minute. At the same time, the tank is being drained at a rate of 5 litres per minute.

- (a) How much acid will there be in the tank after *t* minutes?
- (b) How much acid will there be in the tank after 10 minutes?

Example 16.24

For the differential equation of $\frac{dy}{dx} = x + y$, such that y(2) = 0, use Euler's method with a step value of 0.2 to find an approximate value of y when x = 3, giving your answer to 2 decimal places.

Example 16.25 Given that $\frac{dy}{dx} = \frac{x+1}{xy+2}$ and y = 1 when x = 0, use Euler's method with Step size h = 0.25 to approximate the value of y when x = 1. Give the approximation to 3 significant figures.

Example 16.26

- (a) Show that $\sin \theta \approx \theta$ if θ is close to 0 (θ is in radians).
- (b) Use the approximation from (a) to approximate sin 2°, and compare your approximation to the result you get from your GDC.

Example 16.27

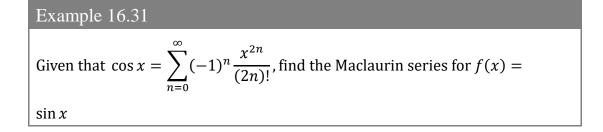
Find the local linear and quadratic approximations of e at x = 0, and graph e^x and the two approximations together.

Example 16.28

Find the Maclaurin polynomials P_0 , P_1 , P_2 , P_3 , and P_n , for e^x and graph the first four together.

Example 16.29		
Find the Maclaurin series for:		
(a) $f(x) = \sin x$	(b) $f(x) = \cos x$	$(c) f(x) = e^x$

Example 16.30
Given that
$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$
, find the Maclaurin series for
 $f(x) = \cos x$



Example 16.32	
Find the Maclaurin series for $f(x) = (1 + x)^k$, where k is a real number.	

Exa	ample 16.33
Fin	d the Maclaurin series for $f(x) = \frac{1}{1+x}$

Example 16.34

Find the Maclaurin series for $g(x) = \frac{1}{1-x}$

Example 16.35	
Find the Maclaurin series for $g(x) = \frac{1}{\sqrt{9-x}}$	

Example 16.36 Find the Maclaurin series for $g(x) = \ln(1+x)$

Example 16.37 Find the Maclaurin series for $g(x) = \arctan x$

Example 16.38

- (a) Find the Maclaurin series for $f(x) = e^{x^2}$
- (b) Hence, find a series for $\int e^{x^2} dx$
- (c) Use the first four terms of the series in (b) to approximate the value of

 $\int_0^1 e^{x^2} \mathrm{d}x.$

Example 16.39		
Find the first three non-zero	terms in the Maclaurin series for:	
(a) $e^{x^2} \sin x$	(b) $\frac{\sin x}{2e^{x^2}}$	

Example 16.40

Use the first three terms of an appropriate power series to estimate $\sqrt[5]{33}$

Example 16.41

Use the power series method to solve the differential equation $\frac{dy}{dx} - y = 0$

Example 16.42

Use the power series method to solve the differential equation y' - xy = 0, y(0) = 2