(1) Which of the following statements is logically equivalent to: "If he studies, he will pass the course." (2010)

- (A) He passed the course; therefore, he studied.
- (B) He did not study; therefore, he will not pass the course.
- (C) He did not pass the course; therefore, he did not study.
- (D) He will pass the course only if he studies.
- (E) None of the above.

Hint: Of these, the contrapositive is the logical equivalent of the original statement.

(2) A point moves in a plane so that its distance from the origin is always twice its distance from

point (1, 1). All such points form (2010)

- (A) a line
- (B) a circle
- (C) a parabola
- (D) an ellipse
- (E) a hyperbola

Hint: $x^2 + y^2 = 4(x^2 - 2x + y^2 - 2y)$

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(3)
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$$y = \begin{cases} |x-2|+4, & x \le 1\\ 4x^2+4, & x > 1 \end{cases}$$

What is the range of the piecewise function? (2011) (2013)

- (A) $y \ge 0$
- (B) y ≥ 5
- (C) $4 \le y \le 8$
- (D) y > 4
- (E) All real numbers
- (4) If $8\sin^2\theta 2\sin\theta = 0$, then what is the smallest positive value of θ ? (2011)
- (A) 7.3°
- (B) 14.5°
- (C) 30°
- (D) 60°
- (E) 75.5°

(5) $P(x) = x^5 + x^4 - 2x^3 - x - 1$ has at most n positive zeros. Then n= (2012) (2014) (A) 0

- (B) 1
- (C) 2
- (D) 3
- (D) 5
- (E) 5

Hint: Descartes' Rule of Sign guarantees at most one positive zero because P(x) has only 1 sign change.

(6) What is the sum of the roots of the equation $(x - \sqrt{2})(x^2 - \sqrt{3}x + \pi) = 0$? (2012)

- (A) -0.315
- (B) -0.318
- (C) 1.414
- (D) 3.15
- (E) 4.56
- Hint: the sum of the roots

(7) The product of 45,454,545,454,545 and 1,234 contains how many digits? (2012) (2010*)

- (A) 14
- (B) 15
- (C) 16
- (D) 17
- (E) 18
- Hint: using log

(8) In a class of 25 students, 80% are passing the class with a grade of C or better. If two students are randomly selected from the class, what is the probability that neither student is passing with a grade of C or better? (2012)

- (A) 0.03
- (B) 0.20
- (C) 0.08
- (D) 0.63
- (E) 0.64
- Hint: the probability with combination

| Test Number | Student's Score | Class Mean | Class Standard Deviation |
|----------------|--------------------|---------------|-----------------------------|
| 1 | 85 | 76 | 3 |
| 2 | 87 | 89 | 7 |
| 3 | 94 | 88 | 4 |
| 4 | 88 | 80 | 6 |
| 5 | 82 | 72 | 5 |

(9) The table above shows a student's record of performance on five tests. On which test did the student rank the highest in relation to the other students in the class? (Assume that the test scores on each test are normal distributed.) (2012)

- (A) Test 1
- (B) Test 2
- (C) Test 3
- (D) Test 4
- (E) Test 5

(10) Two cards are drawn from a regular deck of 52 cards. What is the probability that both will be

- 7s? (2011) (2013)
- (A) 0.149
- (B) 0.04
- (C) 0.012

(D) 0.009

(E) 0.005

Hint: the probability with combination

(11) If the system of equations

$$\begin{cases} ax + by = 2\\ cx + dy = 3 \end{cases}$$

has infinitely solutions, then $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (2013)$

- (A) **-**6
- (B) -1
- (C) 0
- (D) 1
- (E) 6

(12) A pair of dice is tossed 10 times. What is the probability that no 7s or 11s appear as the sum of the sides facing up? (2011) (2013) (2014)

(A) 0.08

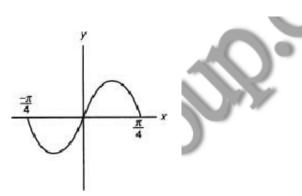
(B) 0.09

(C) 0.11

- (D) 0.16
- (E) 0.24

Hint: There are six ways to get a 7 and two ways to get an 11 on two dice.

(13)



Which of the following could be the equation of one cycle of the graph in the figure above? (2013)

- I. $y = \sin 4x$
- II. $y = \cos(4x \frac{\pi}{2})$
- III. $y = -\sin(4x + \pi)$
- (A) only I
- (B) only I and II
- (C) only II and III
- (D) only II
- (E) I, II, and III

(14) Which of the following quadratic equations has roots 7 + i and 7 - i? (2014)

- (A) $x^2 14x + 49 = 0$
- (B) $x^2 + 14x 48 = 0$
- (C) $x^2 14x + 48 = 0$
- (D) $x^2 14x + 50 = 0$
- (E) $x^2 + 14x + 50 = 0$

(15) If f(x) = 3 sin(πx) + cos(2πx), what is the period of the function? (2013) (2016)
(A) 2
(B) 3
(C) 4
(D) 2π
(E) 3π
Hint: the LCM of the two separate periods

(16) Three pairs of gloves-a red pair, a blue pair, and a green pair-are in a drawer. If gloves are removed at random without returning any to the drawer, what is the minimum number that must be removed in order to guarantee having a matched pair of gloves? (**2017**)

- (A) Two
- (B) Three
- (C) Four
- (D) Five
- (E) Six

(17) For which of the following values of x in the interval $[0, 2\pi]$ is $f(x) = \frac{2}{3\sin(x)+2}$

undefined? (2013) (2017)

- (A) 0.73
- (B) 2.30
- (C) 2.41
- (D) 2.74
- (E) 3.87

(18) If 3x + y = 9, what is the maximum value of x^*y ? (2017)

- (A) $\frac{3}{2}$
- (B) $\frac{27}{4}$
- (C) $\frac{27}{2}$
- (D) $\frac{81}{4}$
- (E) $\frac{81}{2}$

Hint: parabola

(19) What are the rectangular coordinates of the point with polar coordinates (2, 200°)? (2017)
(2013*)
(A) (-1.88, -0.68)

(B) (-0.94, -0.34)
(C) (-0.68, -1.88)
(D) (-0.47, -0.17)
(E) (-0.34, -0.94)

(20) When a company first introduced a new type of cell phone, it initially sold 400 such phone. After 5 weeks, a total of 850 such phones was sold. After 10 weeks, a total of 1,520 such phones was sold. If an exponential regression is used to model the sales of the new type of cell phone, what is the predicted minimum number of weeks it would take to sell a total of 10,000 such phones? (2013) (2017)

- (A) 21
- (B) 24
- (C) 28
- (D) 86
- (E) 107

(21) If $k^2 - 16 < 0$, how many zeros does the function $f(x) = 2x^{2kx} + 8$ have? (2018) (A) 2 (B) 3

- (C) 0
- (0) 0
- (D) 1
- (E) It cannot be determined.

(22) What is units digit of the product of all even integers from 2 to 98, not including 10s? (2018)

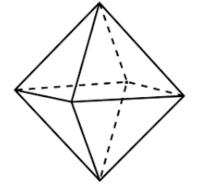
- (A) 2
- (B) 4
- (C) 6
- (D) 8
- (E) 0

(23) How many solutions of θ between $0 \le \theta \le 4\pi$ does $\tan \theta = \cos \theta$? (2018)

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

(24)



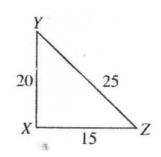


If the above octahedral is sliced by a plane, what is the maximum number of sides a cross section can have? (2018)

- (A) 8
- (B) 6
- (C) 5
- (D) 4
- (E) 3

(25) If $\cos x = -\sqrt{1 - (\sin x)^2}$, then x= (2018)

- (A) all angles
- (B) no angle
- (C) 0
- (D) 2&3 quadrants
- (E) 1&4 quadrants



What is the volume of the solid obtained by the rotating the triangle above about the side of \overline{XY} ?

- (2013) (2019)
- (A) **375**π
- (B) **1,250**π
- (C) **1,500**π
- (D) **1,875**π
- (E) **2,000**π